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The simplest little Higgs

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Abstract

We show that the $SU(3)$ little Higgs model has a region of parameter space in which electroweak symmetry breaking is natural and in which corrections to precision electroweak observables are sufficiently small. The model is anomaly free, generates a Higgs mass near 150 GeV, and predicts new gauge bosons and fermions at 1 TeV.

1 Introduction

The Standard Model Higgs mass suffers from quadratically divergent quantum corrections which destabilize electroweak symmetry breaking. In the presence of these divergences it is unnatural for the Higgs vacuum expectation value and the W and Z masses to be lower than the cutoff by more than a factor of 4π. Turning the argument around: we know the W and Z masses to be around 100 GeV, therefore any natural extension of the Standard Model must contain new physics at or below ~ 1 TeV in order to cancel the divergences. It is exciting that the scale of new physics is within reach of the LHC and possibly also the Tevatron.

Possibilities for this new physics include supersymmetry, technicolor, extra dimensions, or the Little Higgs [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. New physics contributes corrections to precision electroweak observables via tree level exchange of new heavy particles or loop effects. Experiments are so precise that they push the (indirect) lower bounds on new particle masses up to several TeV [11, 12, 13, 14]. This presents a challenge to model building: in order to avoid fine tuning new physics is required near 1 TeV, but particles with TeV scale masses are already severely constrained experimentally.

Recently, Cheng and Low [10] pointed out that little Higgs models can be constructed with a symmetry (T parity) that forbids all tree level contributions from the new physics to electroweak observables while still allowing the loops necessary to cancel divergences. T parity is analogous to R parity in supersymmetric models.

In this paper we point out that the $SU(3)$ little Higgs model proposed by Kaplan and Schmaltz [6] allows a natural solution of the "little hierarchy problem" without T parity. We find that the model has regions of parameter space for which TeV scale particles only couple very weakly to Standard Model fields in tree level interactions. This allows them to hide from precision electroweak measurements while still canceling the divergences to the Higgs mass. We find that new fermion and gauge boson masses as low as 1 TeV are consistent with the data.

In the original $SU(3)$ model [6] anomalies are not canceled in the low energy theory, thus requiring new structure at the cut-off. Here we present a different – anomaly free – choice of fermion representations [15], which requires no spectators, and provides a better fit to precision electroweak measurements. This anomaly free version of the $SU(3)$ model has also recently been UV extended to ~ 50 TeV with another little Higgs theory [16].

In the following section we review the essential ingredients of the $SU(3)$ model. The weak interactions of the Standard Model are enlarged from $SU(2)\times U(1)$ to $SU(3)\times U(1)$ at the TeV scale. The new $SU(3)$ gauge bosons automatically cancel quadratic divergences to the Higgs mass from W and Z loops. Furthermore, in making the top Yukawa coupling $SU(3)$ invariant one introduces a new particle which also cancels the quadratic divergence to the Higgs mass from the top loop. We also determine the Higgs potential and show that top quark loops generate a potential for the Higgs which leads to dynamical electroweak symmetry breaking. Obtaining the correct Higgs VEV and a sufficiently large Higgs mass ($m_H > 114$ GeV) requires inclusion of a tree level potential term similar to the μ term in supersymmetry.

In the third Section we discuss naturalness and precision electroweak constraints. This $SU(3)$ model differs from other little Higgs models in that the Higgs quartic coupling is generated from radiative corrections at low energies. This has the advantage that it automatically avoids quadratically divergent contributions to the Higgs mass from Higgs loops. On the flip side, it implies a small quartic coupling which in turn requires a small Higgs mass parameter to obtain the correct VEV. A small Higgs mass combined with naturalness arguments requires the new physics which cuts off quadratic divergences to be very light. Interestingly, precision data do allow the new states in this model to be light. We find a region in parameter space with new gauge bosons and fermions as light as 1 TeV and only relatively mild tuning of the Higgs mass parameter of order 10% (to be compared with $\sim 2\%$ in minimal supergravity). This mild tuning can be avoided by also implementing collective symmetry breaking in the Higgs potential. In this way one can generate a tree level contribution to the quartic potential at

the cost of additional structure (see e.g. the $SU(4)$ model [6] or a model by Skiba and Terning [8]).

We conclude the paper with a brief discussion of the phenomenology.

2 The model

The underlying prescription for constructing the model is very simple. Enlarge the SM $SU(2)_w \times U(1)_Y$ gauge group to $SU(3)_w \times U(1)_X$ in a minimal way. This entails enlarging $SU(2)$ doublets of the SM to $SU(3)$ triplets, adding $SU(3)$ gauge bosons, and writing $SU(3)$ invariant interactions which reproduce all the SM couplings when restricted to SM fields.

Explicitly, a SM generation is embedded in $(SU(3)_c, SU(3)_w)_{U(1)_X}$ representations

$$\begin{aligned} \Psi_Q = (3,3)_{\frac{1}{3}} \qquad & \Psi_L = (1,3)_{-\frac{1}{3}} \\ d^c = (\bar{3},1)_{\frac{1}{3}} \qquad & e^c = (1,1)_1 \\ 2 \times u^c = (\bar{3},1)_{-\frac{2}{3}} \qquad & n^c = (1,1)_0 \end{aligned} \tag{1}$$

where the triplets Ψ_Q and Ψ_L contain the quark and lepton doublets and u^c, d^c, e^c, n^c are the (charge-conjugated) singlets.[1] There are two u^c fields, one is the SM right-handed up-type quark, the other obtains a large mass with the third components of the triplet Ψ_Q. Similarly, the singlet n^c is the Dirac partner of the third component of Ψ_L.

The symmetry breaking, $SU(3)_w \times U(1)_X \to SU(2)_w \times U(1)_Y$, is achieved with aligned vacuum expectation values for two complex triplet scalar fields

$$\Phi_1, \Phi_2 = (1,3)_{-\frac{1}{3}}. \tag{2}$$

The gauge interactions of the model are uniquely determined by gauge invariance. SM Yukawa couplings and masses for the heavy exotic fermions

[1] This charge assignment for the fermions leaves the $SU(3)_w$ and $U(1)_X$ gauge groups anomalous. The anomalies can be canceled by a Wess-Zumino term. This term is a higher dimensional operator but does not become strongly coupled until the cutoff $\Lambda = 4\pi f$. In order to avoid having to cancel anomalies with spectator fermions at the cutoff we present a different – anomaly free – embedding in the section on fermions.

arise from the couplings

$$
\begin{aligned}
\lambda_1^u u_1^c \Phi_1^\dagger \Psi_Q + \lambda_2^u u_2^c \Phi_2^\dagger \Psi_Q + \lambda^d \frac{d^c \Phi_1 \Phi_2 \Psi_Q}{\Lambda} \\
+ \lambda^n n^c \Phi_1^\dagger \Psi_L + \lambda^e \frac{e^c \Phi_1 \Phi_2 \Psi_L}{\Lambda}
\end{aligned}
\tag{3}
$$

after decomposing the fields into $SU(2)_w \times U(1)_Y$ multiplets. Note that the $SU(3)_w$ indices of the rightmost terms are contracted with epsilon tensors. We will discuss the detailed structure of these couplings when we need them for the potential computations in the next section.

We find it convenient to work with non-linear sigma model fields Φ_i which can be obtained from normal complex triplets with vacuum expectation values f_1 and f_2 by integrating out the radial modes. The non-linear sigma model is more general as it may arise from many different UV completions, the linear sigma model is only one example. 5 of the 10 degrees of freedom in the Φ_i are eaten by the Higgs mechanism when $SU(3)_w$ is broken. We parameterize the remaining degrees of freedom as

$$
\Phi_1 = e^{i\Theta \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \Phi_2 = e^{-i\Theta \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} \tag{4}
$$

where

$$
\Theta = \frac{1}{f} \left[\frac{\eta}{\sqrt{2}} + \begin{pmatrix} 0 & 0 & \multirow{2}{*}{h} \\ 0 & 0 & \\ \multicolumn{2}{c}{h^\dagger} & 0 \end{pmatrix} \right] \quad \text{and} \quad f^2 = f_1^2 + f_2^2 \ . \tag{5}
$$

Here the field h is an $SU(2)_w$ doublet which we identify with the SM Higgs doublet and η is a real scalar field. Their normalizations are chosen to produce canonical kinetic terms.

2.1 Gauge bosons

In addition to the SM gauge bosons our model contains 5 new gauge bosons with masses of order the scale f. The new gauge bosons fill out a complex $SU(2) \times U(1)$ doublet (W'_+, W'_0) with hypercharge $\frac{1}{2}$ and a neutral singlet Z'.

The gauge boson masses are determined from the kinetic terms of the Φ_i

$$|(\partial_\mu + igA^a_\mu T^a - \frac{i}{3} g_x A^x_\mu)\Phi_i|^2 \to tr[(gA^a_\mu T^a - \frac{1}{3} g_x A^x_\mu)^2 \Phi_i \Phi_i^\dagger] \tag{6}$$

To compute the masses it is convenient to use the 3×3 matrix

$$< \Phi_1 \Phi_1^\dagger + \Phi_2 \Phi_2^\dagger >= \begin{pmatrix} <hh^\dagger> & 0 \\ 0 & f^2 \end{pmatrix} \tag{7}$$

expanded out to leading non-trivial order in h/f, and tracing it with the squares of the generators T^a and T^x. In our conventions $<h^T>= (\frac{v}{\sqrt{2}}\, 0)$. To leading order the masses for the $W_\pm$ and the $SU(2)$ doublet of heavy gauge bosons $(W'_\pm, W'_{0,\bar{0}})$ are

$$\begin{aligned} m^2_{W_\pm} &= \frac{g^2}{4} v^2 \\ m^2_{W'_\pm} &= \frac{g^2}{2} f^2 \\ m^2_{W'_0} &= \frac{g^2}{2} f^2 \; . \end{aligned} \tag{8}$$

The mass matrix for the neutral gauge bosons is more complicated. After $SU(3) \times U(1)_X$ breaking the neutral gauge boson corresponding to the diagonal $SU(3)$ generator $T^8 = \frac{1}{\sqrt{3}}\, diag(\frac{1}{2}, \frac{1}{2}, -1)$ and the $U(1)$ generator T^x mix. The mass eigenstates (before taking into account the Higgs VEV) are

$$\begin{aligned} W^3_\mu &= A^3_\mu \\ B_\mu &= \frac{-g_x A^8_\mu + \sqrt{3} g B^x_\mu}{\sqrt{3g^2 + g_x^2}} \\ Z'_\mu &= \frac{\sqrt{3} g A^8_\mu + g_x B^x_\mu}{\sqrt{3g^2 + g_x^2}} \end{aligned} \tag{9}$$

and the hypercharge gauge coupling is

$$g' = g_x / \sqrt{1 + \frac{g_x^2}{3g^2}} \; . \tag{10}$$

After $SU(2) \times U(1)_Y$ breaking the photon remains massless as in the SM, but the Z mixes with the heavy Z' which leads to small deviations from the SM which we discuss in Section 3. The resulting masses are

$$\begin{aligned} m_Z^2 &= \frac{g^2}{4} v^2 (1+t^2) \\ m_{Z'}^2 &= g^2 f^2 \frac{2}{3-t^2} \end{aligned} \tag{11}$$

where $t = g'/g = \tan\theta_W$ and θ_W is the weak mixing angle.

2.2 Fermions

In this section we describe two different embeddings of the quarks and leptons in $SU(3)_w \times U(1)_X$.

Model 1: All three generations carry identical gauge quantum numbers and the $SU(3)_w \times U(1)_X$ gauge group is anomalous. Fermion masses for all generations arise from the Yukawa couplings of equation (3). The SM down type Yukawa matrix is equal to the matrix $\lambda_d f/\Lambda$ which can be seen easily by expanding out $\epsilon\, \Phi_1 \Phi_2/\Lambda \rightarrow \left((\sigma_2 h)^T\ 0\right) f/\Lambda$.

The up-type Yukawa matrices are more interesting as there are 6 quarks of charge 2/3 which mix with each other. In general this leads to flavor changing neutral currents and is dangerous. We assume that one of the matrices λ_i^u is approximately proportional to the unit matrix. This assumption makes the theory completely safe from flavor changing effects and can probably be relaxed somewhat.

We choose the matrix λ_2^u to be proportional to $\mathbf{1}$. λ_1^u must then be hierarchical in order to produce hierarchical SM quark masses, it can be diagonalized by unitary transformations on the fields Ψ_Q and u_1^c. In this basis the 6×6 mass matrix for charge 2/3 quarks decouples into three 2×2 matrices, each describing the mixing of a SM up-type quark (u, c, t) and it's heavy partner (U, C, T). Explicitly, the mass term is

$$(\, u_1^c\ u_2^c) \left(\begin{array}{c} \lambda_1^u \Phi_1^\dagger \\ ---\\ \lambda_2^u \Phi_2^\dagger \end{array} \right) \Big(\Psi_Q \Big) \tag{12}$$

where we suppressed generation indices. After substituting VEVs for the Φ_i the 2×2 submatrices for each generation are

$$\begin{pmatrix} \lambda_1^u <h> f_2/f & \lambda_1^u f_1 \\ -\lambda_2^u <h> f_1/f & \lambda_2^u f_2 \end{pmatrix} \tag{13}$$

Diagonalizing, we find the masses of the up-type quarks and their partners

$$\begin{aligned} m_u &= \lambda_u <h> \\ m_U &= \sqrt{(\lambda_1^u f_1)^2 + (\lambda_2^u f_2)^2} \end{aligned} \tag{14}$$

where we have defined

$$\lambda_u = \lambda_1^u \lambda_2^u \frac{f}{m_U} . \tag{15}$$

For the first two generations $\lambda_1^u \ll \lambda_2^u$, and these expressions further simplify to $m_u = \lambda_1^u <h>$ and $m_U = \lambda_2^u f_2$. Diagonalizing the mass matrices mixes the up-type quarks fields in Ψ_Q with their $SU(2)$ singlet partners by an amount

$$\theta_u = <h> f_1/(f_2 f) \tag{16}$$

which leads to shifts of order θ_u^2 in the W and Z couplings of the up-type quarks and neutrinos.

Model 2: In the second model we cancel the $SU(3)_w$ anomaly by taking different charge assignments for the different generations of quark triplets

$$\begin{array}{rrr} \Psi_{Q^3} = (3,3)_{\frac{1}{3}} & \Psi_{Q^{1,2}} = (3,\bar{3})_0 & \Psi_L = (1,3)_{-\frac{1}{3}} \\ d^{c3} = (\bar{3},1)_{\frac{1}{3}} & 2 \times d^{c1,2} = (\bar{3},1)_{\frac{1}{3}} & e^c = (1,1)_1 \\ 2 \times u^{c3} = (\bar{3},1)_{-\frac{2}{3}} & u^{c1,2} = (\bar{3},1)_{-\frac{2}{3}} & n^c = (1,1)_0 \end{array} \tag{17}$$

where the superscripts label generations. The leptons are unchanged from the previous model. With this new charge assignment all anomalies cancel [15] which makes this model easier to UV complete [16]. Note that now the heavy quarks (T, S, D) are partners of the top, strange and down quarks, respectively. The quark Yukawa couplings stem from operators of the form

$$u^{c\,i} \Phi^\dagger \Psi_{Q^3} + \frac{u^{c\,i} \Phi^\dagger \Phi^\dagger \Psi_{Q^{1,2}}}{\Lambda} + d^{c\,i} \Phi \Psi_{Q^{1,2}} + \frac{d^{c\,i} \Phi \Phi \Psi_{Q^3}}{\Lambda} + \tag{18}$$

where we have suppressed indices labeling the Φ_i and the two copies of conjugate fields. As in the previous model these operators allow general 3×3 Yukawa matrices for up and down quarks and leptons. The diagonalization of mass matrices is different from the previous model as there is only one new up-type quark and two new down-type quarks and associated mixing angles θ_d with the light quarks. In order to avoid flavor changing neutral currents the mass matrix of heavy partners needs to be sufficiently well aligned with the quark mass matrices to avoid flavor changing neutral currents. We expect the constraints from flavor physics to be interesting and non-trivial [17] but a detailed study is beyond the scope of this paper.

Note that in both models the mixing of light fermions with their partners generates a coupling of the W and Z gauge bosons to a single SM fermion and it's heavy partner proportional to $\theta_{u,d}$. This opens the interesting possibility of single U, D production from fusion of weak gauge bosons with SM quarks (e.g. $d + W \to U,\ u + Z \to U$).

2.3 Scalars

The two scalar triplets Φ_i which are responsible for $SU(3) \times U(1) \to SU(2) \times U(1)$ breaking contain 10 real degrees of freedom. 5 are eaten by the $SU(3)$ gauge bosons with TeV scale masses, 4 form the SM Higgs doublet h and one is a real scalar field η. Since we did not include an operator which gives a quartic coupling for the Higgs, this must be generated dynamically. Explicitly, the radiative corrections should produce the standard model Higgs potential

$$V = m^2 h^\dagger h + \lambda (h^\dagger h)^2 \tag{19}$$

Electroweak symmetry breaking requires m^2 negative and of order of the electroweak scale, and the LEP bound on the Higgs mass requires $\lambda \gtrsim 0.11$. We now discuss the form of the radiative contributions to the potential. Above the scale f the $SU(3)$ gauge symmetry is unbroken and the potential is best described in terms of the $SU(3)$ multiplets Φ_i, and it is easy to see that the most general potential is a function of the only gauge invariant which

depends on the Higgs, $\Phi_1^\dagger\Phi_2$. At the scale f, the $SU(3)$ partners of fermions and gauge bosons obtain masses, and the theory matches onto the standard model. Below f the Higgs potential receives the usual radiative corrections from top quark and gauge loops.

We first discuss the potential generated above the scale f. The top Yukawa couplings and gauge couplings preserve a $U(1)$ symmetry under which Φ_1 and Φ_2 have opposite charges. Therefore the lowest dimensional operator which can be radiatively induced is $|\Phi_1^\dagger\Phi_2|^2$. This operator is already generated at one loop but only with a logarithmic divergence. Its contribution to the Higgs mass is of order $f^2/(16\pi^2) \sim m_W^2$. The symmetry forbids any quadratically divergent contributions from gauge or Yukawa couplings.

As can be seen from the explicit formulae below, the radiatively generated potential alone generates a Higgs "soft mass squared" which is somewhat too large. Therefore we also include a tree level "μ" term which will partially cancel the Higgs mass. It explicitly breaks the spontaneously broken global $U(1)$ symmetry and gives a mass to the would-be Nambu-Goldstone boson η

$$V_{tree} = \mu^2\Phi_1^\dagger\Phi_2 + h.c. \to \mu^2\frac{f^2}{f_1 f_2}(h^\dagger h + \frac{1}{2}\eta^2) - \frac{1}{12}\frac{\mu^2 f^4}{f_1^3 f_2^3}(h^\dagger h)^2 + \dots \qquad (20)$$

Since the operator contains a Higgs mass, μ must be near the weak scale m_W. The mass scale μ is radiatively stable because this is the only $U(1)$ breaking operator in the theory.

We compute the radiatively generated one-loop potential from gauge and Yukawa interactions using the formalism of Coleman and Weinberg [18]. Details of the calculation are given in an Appendix. The corrections to the potential Eq. (19) contain a mass squared δm^2 and a quartic $\delta\lambda$

$$\delta m^2 = \frac{-3}{8\pi^2}\left[\lambda_t^2 m_T^2 Log\left(\frac{\Lambda^2}{m_T^2}\right) - \frac{g^2}{4}m_{W'}^2 Log\left(\frac{\Lambda^2}{m_{W'}^2}\right) - \frac{g^2}{8}(1+t^2)m_{Z'}^2 Log\left(\frac{\Lambda^2}{m_{Z'}^2}\right)\right]$$

$$\delta\lambda = \frac{|\,\delta m^2|}{3}\frac{f^2}{f_1^2 f_2^2} + \frac{3}{16\pi^2}\left[\lambda_t^4 Log\left(\frac{m_T^2}{m_t^2}\right) - \frac{g^4}{8}Log\left(\frac{m_{W'}^2}{m_W^2}\right) - \frac{g^4}{16}(1+t^2)^2 Log\left(\frac{m_{Z'}^2}{m_Z^2}\right)\right]$$

Assuming that there are no large direct contributions to the potential from physics at the cutoff we have

$$V_{total} = (\mu^2 \frac{f^2}{f_1 f_2} + \delta m^2) h^\dagger h + (-\frac{1}{12} \frac{\mu^2 f^4}{f_1^3 f_2^3} + \delta\lambda)(h^\dagger h)^2 \tag{21}$$

Note that the radiative contribution to the Higgs mass from the top loop is negative while the contribution to the quartic is positive. Thus we have radiative electroweak symmetry breaking and stability of the Higgs potential.

The potential depends on a number of free parameters, before analyzing it further we first determine what ranges for the parameters are reasonable by studying the constraints from precision electroweak data.

3 Electroweak constraints and naturalness

The model predicts small changes to the masses and couplings of the Z from Z-Z' mixing and to the fermion couplings from q-Q mixing. Both types of corrections scale as v^2/f^2 and decouple as we take f large. Naturalness arguments prefer f to be as near to the TeV scale as possible, therefore a detailed a study of the size of the corrections is needed. This study has been performed for a model [8] which closely related to our model 1. We quote some of the results and add constraints from LEP II.

To start, note that the numerically most significant contribution to the Higgs mass comes from the top loop, therefore naturalness arguments primarily lead to an upper bound on the mass of the heavy partner of the top quark

$$m_T = \sqrt{(\lambda_1^t f_1)^2 + (\lambda_2^t f_2)^2} \; . \tag{22}$$

On the other hand, deviations from the standard model in precision electroweak physics stem mostly from mixing with the Z' and from four fermion operators mediated by the Z'. Thus precision electroweak physics implies a lower bound on the mass of the Z'

$$m_{Z'}^2 = \frac{2g^2}{3-t^2}(f_1^2 + f_2^2) \tag{23}$$

We see that it is possible to lower the mass of the top partner while keeping the Z' mass fixed by going to a region in parameter space in which the f_i are different. Then the Z' mass is dominated by the larger of the two f_i whereas m_T can be made smaller by reducing the corresponding Yukawa coupling λ_i^t.

To reduce the number of parameters we determine the λ_i^t such that the T-mass is minimized for given scales f_i and top Yukawa coupling Eq. (14). This gives the values

$$\lambda_1 = \sqrt{2}\lambda_{top}\frac{f_2}{f} \qquad \lambda_2 = \sqrt{2}\lambda_{top}\frac{f_1}{f} \qquad m_T = 2\lambda_{top}\frac{f_1 f_2}{f} \tag{24}$$

The T-mass is not very sensitive to this precise choice, i.e. it is not fine-tuned.

In the following, we imagine the scales f_i to differ by a factor of a few with $f_2 > f_1$. This choice reduces fermion mixing Eq. (16) sufficiently, such that no significant deviations from the standard model arise, and we ignore fermion mixing for the precision electroweak analysis.

The largest deviations from the standard model derive from tree level Z-Z' mixing and Z' induced four fermion operators. We find the custodial $SU(2)$ symmetry violating shift in the Z mass

$$\delta\rho \equiv \alpha T = \frac{1}{8}\frac{v^2}{f^2}(1-t^2)^2 \ , \tag{25}$$

the modified Z-couplings

$$\frac{e}{c_w s_w} Z^\mu \left[J^3_\mu - s_w^2 J^Q_\mu - \frac{v^2}{8f^2}(1-t^2)(\sqrt{3}J^8_\mu + t^2 J^Y_\mu) \right] \ , \tag{26}$$

and the Z' induced four fermion operators

$$\delta\mathcal{L} = -\frac{(\sqrt{3}J^8_\mu + t^2 J^Y_\mu)^2}{4f^2} \ . \tag{27}$$

Here $s_w = sin\,\theta_w$, $c_w = cos\,\theta_w$, and the fermion currents J_μ are defined such that the neutral gauge bosons couple as $gA_3J^3 + gA_8J^8 + g_xA_xJ^x$, and $J^Y \equiv J^x - J^8/\sqrt{3}$, $J^Q \equiv J^3 + J^Y$. Note that these formulae apply to both fermion embeddings. As is customary we use the experimentally measured values for α_{em}, s_w, G_F to fix the parameters g, s_w, v in our lagrangian.

We begin by looking at the bound implied by the Z-mass shift which corresponds to a non-vanishing T parameter. Applying the 95% confidence limit $T < 0.15$ (for a light Higgs) [11] we obtain $f \gtrsim 2.0$ TeV in either model.

LEP II data constrain the possible contribution to e^+e^- scattering from four fermion operators [12]. We find that the best constraint comes from considering the operator $(\bar{e}\gamma^\mu\gamma^5 e)^2$ contained in Eq. (27) which gives $f \gtrsim 2.0$ TeV at the 95% confidence level in both models.

The strongest constraint in the analysis of [8] is due to new contributions to atomic parity violation, both from the shift in the Z coupling as well as the four Fermi operators. Since the couplings of the first generation quarks to the Z and Z' differ in the two models we find different limits. In model 1 the magnitude of the predicted "weak charge" of Cesium is increased proportional to $\frac{v^2}{f^2}$. Using the experimental value [11, 19] $Q_W^{exp} = -72.69 \pm .48$ and the Standard Model prediction $Q_W^{SM} = -73.19 \pm .03$ we find a strong limit $f \gtrsim 3.9$ TeV at 95% confidence level. In model 2 the weak charge is reduced due to the new interactions. This improves the fit and leads to a bound $f \gtrsim 1.7$ TeV (at 95% confidence).

The results for model 1 are similar to the results of [8] who performed a fit to all precision electroweak measurements and find $f > 3.3$ TeV at 95% confidence, and $f \gtrsim 2.5$ TeV if the Cesium APV constraint (which depends on difficult to quantify systematic errors in atomic physics calculations) is dropped.

We summarize that there is significant tension between precision electroweak constraints and naturalness in model 1. Model 2 avoids all constraints in the region of parameter space with $f \sim f_2 \gtrsim 2$ TeV and f_1 somewhat smaller. For example, picking the "golden" point, $f_1 = 0.5$ TeV and $f_2 = 2$ TeV, we find

$$\begin{aligned} m_T &= 1.0\,\mathrm{TeV} \qquad m_D = m_S = 0.7\,\mathrm{TeV} \\ m_{Z'} &= 1.1\,\mathrm{TeV} \qquad\qquad m_{W'} = 0.9\,\mathrm{TeV} \end{aligned} \tag{28}$$

For this point we can now also determine the approximate Higgs mass from Eq. (21). Fixing the Higgs VEV at 246 GeV, the cutoff $\Lambda = 5$ TeV

($\sim 4\pi f_1$), we find $m_{Higgs} = 140$ GeV. The amount of fine tuning can be defined using the sensitivity of the squared Higgs VEV v^2 to the parameter μ^2, i.e. $\frac{\mu^2}{v^2}\frac{\partial v^2}{\partial \mu^2}$. We find a sensitivity of about 10 for the parameter choice above which corresponds to 10% tuning.

4 Conclusions

We have shown that the $SU(3)$ simplest little Higgs model has a natural region of parameter space in which precision electroweak constraints are satisfied and electroweak symmetry breaking only requires mild tuning of order 10% (to be compared with $\sim 2\%$ for the MSSM).

It would be interesting to explore flavor changing effects mediated by the new particles. We expect nontrivial constraints because the couplings of new physics to the third generation differs from couplings to the first two generations.

LHC phenomenology for this model is exciting as it promises new gauge bosons and fermions near 1 TeV, both within reach of the LHC [20, 21]. Easiest to produce and detect is the Z' which can be singly produced and has a significant branching ratio into lepton pairs. Heavy quarks can be produced singly via fusion of a u or d quark with a weak gauge boson (in analogy with single top production) or in pairs from strong interactions.

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A Appendix: Coleman-Weinberg potential

We compute the one-loop radiatively generated Higgs potential using the method of Coleman and Weinberg [18]. One substitutes the Higgs by its vacuum expectation value and computes the vacuum energy at one loop as a function of the particle masses. Since the particle masses depend on the Higgs expectation value, this determines the Higgs dependence of the vacuum energy: the Higgs potential. At one loop we only need to compute loops with no interactions, summed over all the fields in the theory.

A.1 Fermion contribution

The only numerically relevant contribution from fermion loops is due to the top quark and its partner. To compute it we need to determine the top quark mass matrix in the presence of $SU(2)_w$ breaking. The top Yukawa coupling comes from

$$\mathcal{L}_{top} = (\lambda_1 t_1^c \Phi_1^\dagger + \lambda_2 t_2^c \Phi_2^\dagger)\ \Psi_T\ . \tag{29}$$

For the CW potential we need the hermitian mass squared matrix

$$M_{tT}^2 = \begin{pmatrix} \lambda_1 \Phi_1^\dagger \\ ---- \\ \lambda_2 \Phi_2^\dagger \end{pmatrix} \left(\begin{array}{c|c} \lambda_1 \Phi_1 & \lambda_2 \Phi_2 \end{array} \right) = \begin{pmatrix} \lambda_1^2 \Phi_1^\dagger \Phi_1 & \lambda_1 \lambda_2 \Phi_1^\dagger \Phi_2 \\ \lambda_2 \lambda_1 \Phi_2^\dagger \Phi_1 & \lambda_2^2 \Phi_2^\dagger \Phi_2 \end{pmatrix} \tag{30}$$

Expanding the Φ_i by using Eq. [4] and diagonalizing we find the masses squared of the t and T quarks to fourth order in the Higgs VEV. We use the notation m_t^2 for the leading order (in v^2/f^2) top mass and $m_{t,4}^2$ for the 4-th order expression.

$$\begin{aligned} m_{t,4}^2 &= \lambda_t^2 < h^\dagger h > - \left[\frac{1}{3} \lambda_t^2 \frac{f^2}{f_1^2 f_2^2} - \frac{\lambda_t^4}{m_T^2} \right] < h^\dagger h >^2 \\ m_{T,4}^2 &= m_T^2 - m_{t,4}^2 \end{aligned} \tag{31}$$

where we have used

$$\begin{aligned} m_T^2 &= \lambda_1^2 f_1^2 + \lambda_2^2 f_2^2 \\ \lambda_t &= \lambda_1 \lambda_2 \frac{f}{m_T} \end{aligned} \tag{32}$$

We use the Coleman-Weinberg formula for fermions with mass matrix M_f

$$V_{fermion} = -\frac{N_c}{16\pi^2}\Lambda^2 tr[M_f^2] + \frac{N_c}{16\pi^2} tr[M_f^4 Log\left(\frac{\Lambda^2}{M_f^2}\right)] \tag{33}$$

and see that the quadratic divergence cancels. The Log-divergent piece gives

$$\begin{aligned} V_2 &= -\frac{3}{8\pi^2}\lambda_t^2 m_T^2 Log\left(\frac{\Lambda^2}{m_T^2}\right)\ h^\dagger h \\ V_4 &= \frac{3}{16\pi^2}\left[\lambda_t^4 Log\left(\frac{m_T^2}{m_t^2}\right) + \frac{2}{3}\frac{f^2}{f_1^2 f_2^2}\lambda_t^2 m_T^2 Log\left(\frac{\Lambda^2}{m_T^2}\right)\right]\ (h^\dagger h)^2 \end{aligned} \tag{34}$$

The form of the Logs is suggestive toward an effective field theory interpretation. The $Log\,(\Lambda^2/m_T^2)$ arises from renormalization above the T mass where the theory is $SU(3)$ symmetric. Therefore an $SU(3)$ symmetric potential is generated

$$\begin{aligned} V_{SU(3)} &= \frac{3}{8\pi^2}\,\lambda_1^2\lambda_2^2|\Phi_1^\dagger\Phi_2|^2 Log\left(\frac{\Lambda^2}{\mu^2}\right) + const. \\ &= \frac{3}{8\pi^2}\left[-\lambda_t^2 m_T^2\, h^\dagger h + \frac{1}{3}\lambda_t^2\frac{m_T^2 f^2}{f_1^2 f_2^2}(h^\dagger h)^2 + \dots\right] Log\left(\frac{\Lambda^2}{m_T^2}\right) + const. \end{aligned} \tag{35}$$

where in the last line we replaced $\mu^2 \to m_T^2$. Below m_T, only the top quark remains in the effective theory and produces its usual contribution to the quartic coupling proportional to $Log\,(m_T^2/m_t^2)$.

A.2 Gauge boson contribution

The gauge boson masses stem from the kinetic terms of the Φ_i

$$|(\partial_\mu + igA_\mu^a T^a - \frac{i}{3}g_x A_\mu^x)\Phi_i|^2 \to tr[(gA_\mu^a T^a - \frac{1}{3}g_x A_\mu^x)^2 \Phi_i \Phi_i^\dagger] \tag{36}$$

We begin by computing the matrix

$$\Phi_1\Phi_1^\dagger + \Phi_2\Phi_2^\dagger = \begin{pmatrix} hh^\dagger + (hh^\dagger)^2(1/f^2 - f^2/(3f_1^2 f_2^2)) & 0 \\ 0 & f^2 - h^\dagger h - (h^\dagger h)^2(1/f^2 - f^2/(3f_1^2 f_2^2)) \end{pmatrix} \tag{37}$$

to the relevant order. We then trace this matrix with the squares of the gauge generators and find the masses for the W's and an $SU(2)$ doublet of heavy gauge bosons $(W'_{\pm}, W'_{0,\bar{0}})$

$$
\begin{aligned}
m^2_{4,W_\pm} &= \frac{g^2}{4}v^2[1+\frac{v^2}{2f^2}(1-\frac{1}{3}\frac{f^4}{f_1^2 f_2^2})] \\
m^2_{4,W'_\pm} &= \frac{g^2}{2}f^2 - m^2_{4,W_\pm} \\
m^2_{4,W'_0} &= \frac{g^2}{2}f^2
\end{aligned} \tag{38}
$$

The neutral gauge boson masses are more complicated because the gauge bosons corresponding to the $SU(3)$ generator $T^8 = \frac{1}{2\sqrt{3}}\, diag(1,1,-2)$ and the $U(1)$ generator T^x mix. The mass eigenstates before $SU(2)$ breaking are

$$
\begin{aligned}
W^3_\mu &= A^3_\mu \\
B_\mu &= \frac{-g_x A^8_\mu + \sqrt{3} g B^x_\mu}{\sqrt{3g^2+g_x^2}} \\
Z'_\mu &= \frac{\sqrt{3} g A^8_\mu + g_x B^x_\mu}{\sqrt{3g^2+g_x^2}}
\end{aligned} \tag{39}
$$

and the hypercharge gauge coupling is $g' = g_x/\sqrt{1+g_x^2/3g^2}$.

After $SU(2)\times U(1)_Y$ breaking the photon remains massless as in the SM, however the Z mixes with the Z' which leads to deviations from the SM. Using $t = g'/g = \tan\theta_W$ where θ_W is the weak mixing angle we write the masses for the Z and Z' as

$$
\begin{aligned}
m^2_{4,Z} &= \frac{g^2}{4}v^2(1+t^2)[1+\frac{v^2}{2f^2}(1-\frac{1}{3}\frac{f^4}{f_1^2 f_2^2}) - \frac{v^2}{8f^2}(1-t^2)^2] \\
m^2_{4,Z'} &= \frac{g^2}{2}f^2\frac{4}{3-t^2} - m^2_{4,Z}
\end{aligned} \tag{40}
$$

We insert these masses into the Coleman-Weinberg formula

$$
V_{gauge} = \frac{3}{32\pi^2}\Lambda^2 tr[M_g^2] - \frac{3}{64\pi^2} tr[M_g^4 Log\left(\frac{\Lambda^2}{M_g^2}\right)] \tag{41}
$$

to obtain the contribution to the Higgs potential. As in the fermion case the Higgs dependence in the quadratically divergent term cancels. Summing over $W_\pm$, $W'_\pm$, and W'_0 the Log-divergent term gives

$$\begin{aligned} V_2 &= \frac{3}{32\pi^2} g^2 m_{W'}^2 Log\left(\frac{\Lambda^2}{m_{W'}^2}\right)\ h^\dagger h \\ V_4 &= -\frac{3}{128\pi^2} g^4 \left[Log\left(\frac{m_{W'}^2}{m_W^2}\right) + \frac{2}{3}\frac{f^4}{f_1^2 f_2^2} Log\left(\frac{\Lambda^2}{m_{W'}^2}\right)\right]\ (h^\dagger h)^2 \end{aligned} \tag{42}$$

and from the Z and Z' we find

$$\begin{aligned} V_2 &= \frac{3}{64\pi^2} g^2 (1+t^2) m_{Z'}^2 Log\left(\frac{\Lambda^2}{m_{Z'}^2}\right)\ h^\dagger h \\ V_4 &= -\frac{3}{256\pi^2} g^4 \left[(1+t^2)^2 Log\left(\frac{m_{Z'}^2}{m_Z^2}\right) + \frac{8}{3}\frac{1+t^2}{3-t^2}\frac{f^4}{f_1^2 f_2^2} Log\left(\frac{\Lambda^2}{m_{Z'}^2}\right)\right] (h^\dagger h)^2 \end{aligned} \tag{43}$$

Finally, summing contributions from both top and gauge sectors we have

$$\delta V = \delta m^2 h^\dagger h + \delta\lambda (h^\dagger h)^2 \tag{44}$$

where

$$\delta m^2 = \frac{-3}{8\pi^2}\left[\lambda_t^2 m_T^2 Log\left(\frac{\Lambda^2}{m_T^2}\right) - \frac{g^2}{4} m_{W'}^2 Log\left(\frac{\Lambda^2}{m_{W'}^2}\right) - \frac{g^2}{8}(1+t^2) m_{Z'}^2 Log\left(\frac{\Lambda^2}{m_{Z'}^2}\right)\right]$$

and

$$\begin{aligned} &\delta\lambda = -\frac{m^2}{3}\frac{f^2}{f_1^2 f_2^2} + \frac{3}{16\pi^2} \times \\ &\left[\lambda_t^4 (Log\left(\frac{m_T^2}{m_t^2}\right) - \frac{1}{2}) - \frac{g^4}{8}(Log\left(\frac{m_{W'}^2}{m_W^2}\right) - \frac{1}{2}) - \frac{g^4}{16}(1+t^2)^2 (Log\left(\frac{m_{Z'}^2}{m_Z^2}\right) - \frac{1}{2})\right] \end{aligned}$$

where in the last line we have also restored the finite terms which one obtains from expanding out the masses in the logarithms in the Coleman-Weinberg formula in terms of the Higgs field. In addition there are finite cut-off dependent terms in the CW potential $tr M^4 [Log(\Lambda^2/M^2) + 3/2]$, they can easily be included by redefining $\Lambda \to 2.1\,\Lambda$ in these formulas.

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